COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

SPECTRAL ELEMENT INVESTIGATION OF RADIAL FLOW STABILITY

Jáuber C. Oliveira
Universidade Federal de Santa Catarina, Departamento de Informática e Estatística
Cx. P. 476 - 88040-900 - Florianópolis, SC, Brasil
Cristina H. Amon
Department of Mechanical Engineering,
Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA

Abstract. *Direct numerical simulations are performed to investigate the nonlinear stability and flow evolution of two- and three- dimensional radial flows between parallel stationary disks. The spectral element method is employed for the spatial discretization of the unsteady, incompressible Navier-Stokes equations. Finite amplitude perturbations are introduced at the inlet of the domain to emulate the experimental situation where such finite-amplitude disturbances are convected from upstream sources. It is found that axisymmetric models fail to predict the flow physics for Re > 200, due to the existence of a secondary flow which cannot be captured by two-dimensional simulations. By predicting the flow responses to infinitesimally small perturbations, a characteristic frequency is obtained for the least-stable mode corresponding to the time-evolution of the flow towards steady state. Oscillatory perturbations are then imposed at the inlet of the three-dimensional domain with a frequency matching the least-stable frequency of the decaying perturbations. This tuned oscillatory forcing of the flow induces increase in the circulation and enhancement in mixing of the secondary flow. Once this forcing is turned off, the flow remains unsteady with an initial stage of growing oscillations, leading either to a self-sustained oscillatory flow or to a sub-critical transition.*

Keywords: Spectral element, Nonlinear instabilities, Radial flows, Oscillatory perturbations.

1. INTRODUCTION

Direct Numerical Simulations (DNS) are becoming increasingly feasible and powerful regarding the potential to predict and resolve more complex geometries and higher Reynolds number flows. Numerical methods have evolved in parallel with computer hardware development and methods with origin in the so-called *spectral methods* (see Canuto et al., 1988; Gottlieb and Orszag, 1977) have become an important tool for performing stability analysis. This is due to the fact that numerical simulations to investigate the stability of basic flows require very high accuracy in order to provide reliable data. This requirement matches the fundamental characteristics of spectral and spectral-element methods (SEM). While spectral methods use global functions, SEM (Patera, 1984) combine the accuracy of spectral methods with the flexibility of the finite element methods to handle complex geometries. Amon (1993) extended the methodology for three-dimensional transitional flows and developed the Spectral Element-Fourier Method (SEFM). The high accuracy characterizing these methods is basically due to the use of local (global for *pure* spectral methods) expansions in terms of eigenfunctions from singular Sturm-Liouville problems, which are known to provide *exponential convergence* (as opposed to *algebraic convergence*) if the solutions turns out to be smooth enough. A deeper mathematical reason for such performance comes from the fact that such techniques address the problem of getting *uniform convergence* as opposed to the usual $\mathbf{L}^2$-type of convergence that is common in finite element methodologies.

Radial flows are usually modeled by a relatively simple geometry. However, strong adverse pressure gradients cause flow separation at low Reynolds numbers. As a result, the flow configuration is known to be complex due to the presence of multiple recirculation zones and the unstable characteristics of the flow. Therefore, a thorough stability analysis of radial flows requires the use of accurate tools. An additional fact serves as an example of the challenges to study such flows: even at the Stokes regime, for which the flow is always stable, the basic flow is known to be *non-parallel*. Therefore, classical hydrodynamic stability theory (Drazin and Reid, 1981) concepts of parallel or quasi-parallel flow are not applicable. We refer to Langer (1991), Langer, Prata and Ferreira (1990), Oliveira (1992), Oliveira, Prata and Langer (1993), Peters (1994), Oliveira and Amon (1996), Oliveira (1997) for a review on previous stability analyses of radial flows.

Based on these ideas, the main objective of the investigation presented in this paper is to exploit the feasibility of Direct Numerical Simulations (DNS) with the Spectral Element Method (Patera, 1984; Amon, 1993) to investigate flow instabilities on radial flows governed by the three-dimensional time-dependent Navier-Stokes equations. Specifically, DNS provide information on the range of Reynolds numbers for which the two-dimensional (axisymmetric) models are physically realistic as well as on the behavior of instabilities triggered by perturbations introduced into the three-dimensional basic flow. Finally, our aim is to characterize the onset of transition to turbulence in radial flows.

Details on formulation and methodology are presented in Section 2. The third section contains discussions on the main results obtained. The fourth and last section summarizes our conclusions.

2. FORMULATION AND METHODOLOGY

To attain the goals stated in the previous section, we investigate the stability of the three-dimensional flow between two parallel stationary disks through computational models involving the full domain between the two parallel disks, model (3D-1), or truncations of this model.

The formulation of the problem is based on the following assumptions. We assume that the fluid is Newtonian and the properties (density and viscosity) are constant. The governing equations are those associated with the conservation of mass and linear momentum. The Reynolds number for all models is defined by $Re = Q/(2 \pi \nu s)$, where Q is the flow rate at the radial section $r = r_1$ (entrance of the computational domain), ν is the kinematic viscosity and s is the distance between the two parallel disks. The Stokes flow exact solution (Oliveira, 1997) is prescribed for the radial velocity distribution at the inlet of the computational domain (at $r_1 = 0.5$). The remaining boundary conditions are: no slip at the upper ($z = 1$) and lower ($z = 0$) disks, and a stress-free boundary condition prescribed at the exit of the domain ($r_2 = 10$).

The remaining of this section deals with a more detailed description of the non-dimensional problem considered in the simulations. We choose to present the formulation for the full three-dimensional model 3D-1. We define the following non-dimensional variables:

$$u = \frac{U}{Ve},\ v = \frac{V}{Ve},\ w = \frac{W}{Ve},\ p = \frac{P}{\rho Ve^2},\ x = \frac{X}{s},\ y = \frac{Y}{s},\ z = \frac{Z}{s},\ t = \frac{T}{s / Ve} \tag{1}$$

where U, V, W, P denote the dimensional velocity components and pressure. X, Y and Z correspond to the dimensional coordinates in the X-Y plane and axial coordinate, respectively; x, y and z are the non-dimensional variables corresponding to X, Y and Z; T and t denote the non-dimensional and dimensional times, respectively; u, v and w denote the components of the non-dimensional velocity vector; p represents the non-dimensional pressure. Ve is the mean velocity at the entrance of the computational domain.

The governing equations are as follows,

$$\nabla \cdot \mathbf{v} = 0 \tag{2}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \left(\frac{r_1}{\mathrm{Re}} \right) \nabla^2 \mathbf{v} \tag{3}$$

where $\mathbf{v} = u\ \mathbf{i} + v\ \mathbf{j} + w\ \mathbf{k}$.

The domain of the problem is $\Omega := \left\{ (x, y, z) \subset \mathcal{R}^3 \mid \frac{1}{4} < x^2 + y^2 < 100,\ z \in (0,1) \right\}$ and the boundary conditions are:

$$u = \frac{12(z - z^2)}{x},\ v = \frac{12(z - z^2)}{y},\ w = 0\ \ for\ (x,\ y,\ z) \in Ci \tag{4}$$

$$-p + 2\mu \frac{\partial\ u_n}{\partial\ n} = T_n\ \ for\ (x, y, z) \in Co \tag{5}$$

$$u = v = w = 0\ \ for\ z = 0\ and\ z = 1;\ \forall x, y \in \Omega \tag{6}$$

where $Ci := \left\{ (x, y, z) \subset \mathcal{R}^3 \mid x^2 + y^2 = \frac{1}{4},\ z \in [0,1] \right\}$ and

$Co := \{ (x, y, z) \subset \mathrm{R}^3 \mid x^2 + y^2 = 100,\ z \in (0,1) \}$.

Henceforth, the non-dimensional inner and outer radii are constant: r_1=0.5 and r_2=10.

The spatial discretization is based on the Spectral Element Method (SEM). The computational domain is partitioned into hexahedral macro-elements, each isoparametrically mapped from the physical space to the local coordinate system $(-1, 1)^3$. The geometry, pressure and velocity are represented by high-order tensor-product Lagrangian interpolants through Gauss-Lobatto Legendre collocation points. Velocity and pressure are solved on staggered meshes (velocity on a Gauss-Legendre Lobatto, while the pressure on a Gauss-

Legendre mesh). The temporal discretization is based on an explicit treatment of the nonlinear, convective term by a third-order Adams-Bashforth scheme. This term is the only source of stability conditions for the numerical scheme (CFL condition number). The viscous-diffusive term is treated implicitly by a first-order Crank-Nicholson, multi-step scheme. The solution procedure is based on the Uzawa algorithm.

The procedure to analyze numerically the stability of a certain basic flow field is the following. We introduce artificial perturbations at the entrance of the domain so as to mimic the situation where free-stream perturbations penetrate the domain, disturbing the flow field. After such artificial initial perturbations are imposed, the numerical simulations are executed to analyze the response of the flow. As a result, the temporal evolution and spatial distribution of the *disturbance field* is obtained. Further analysis of such data provides the information on the stability of the basic flow field.

Results obtained with different meshes and truncated domains guide us in the construction of computational meshes which balance good accuracy with computational feasibility. For details on this part of the investigation, we refer to Oliveira (1997).

3. RESULTS

As opposed to a purely *radial* (axisymmetric) *flow*, we call *secondary flow* the flow in the azimuthal direction which we observe to coexist with the radial flow at a Reynolds number above 200 (Oliveira and Amon, 1996).

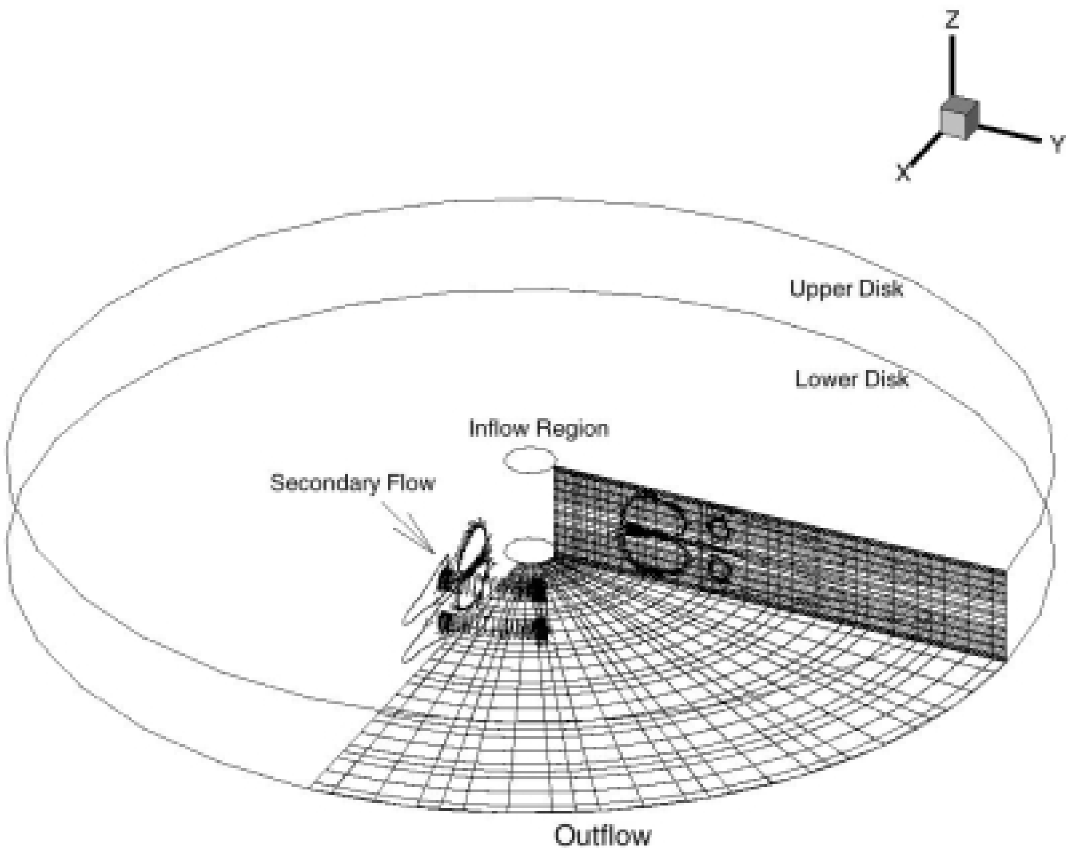

Figure 1: Three-dimensional disturbance streamtraces for Re = 50.

To investigate the nonlinear stability mechanisms in radial flows, we impose two types of perturbations into the numerically obtained basic flow fields: three-dimensional infinitesimally small perturbations and finite perturbations. *Infinitesimally small perturbations* are naturally generated by round-off errors, while *finite perturbations* are imposed at the entrance of the computational domain. It is of interest to know how the flow field will respond to such forcing. The finite amplitude perturbations are o*scillatory perturbations* with frequency matching the characteristic frequency observed in the least stable modes of time-decaying oscillations present in the temporal evolution of the flow under small perturbations.

Figure 1 presents a three-dimensional secondary flow in the disturbance field for Re = 50, a result of introducing a small vortex at the entrance of the domain and then computing the response of the flow to such initial condition. The disturbance response, though decaying in time, provides information on how the flow would loose axisymmetry at higher Reynolds numbers. This was indeed verified for Re = 200 (Oliveira and Amon, 1996) where the flow is fully three-dimensional due to the presence of secondary flows.

The isolines shown in Fig. 1 represent the three-dimensional flow field and are generated by a predictor-corrector integration algorithm based on the velocity vector to determine at every location **x** which path is followed by fluid particles. If the flow field is stationary, such isolines coincide with the streamlines. Henceforth, we call these isolines *streamtraces*.

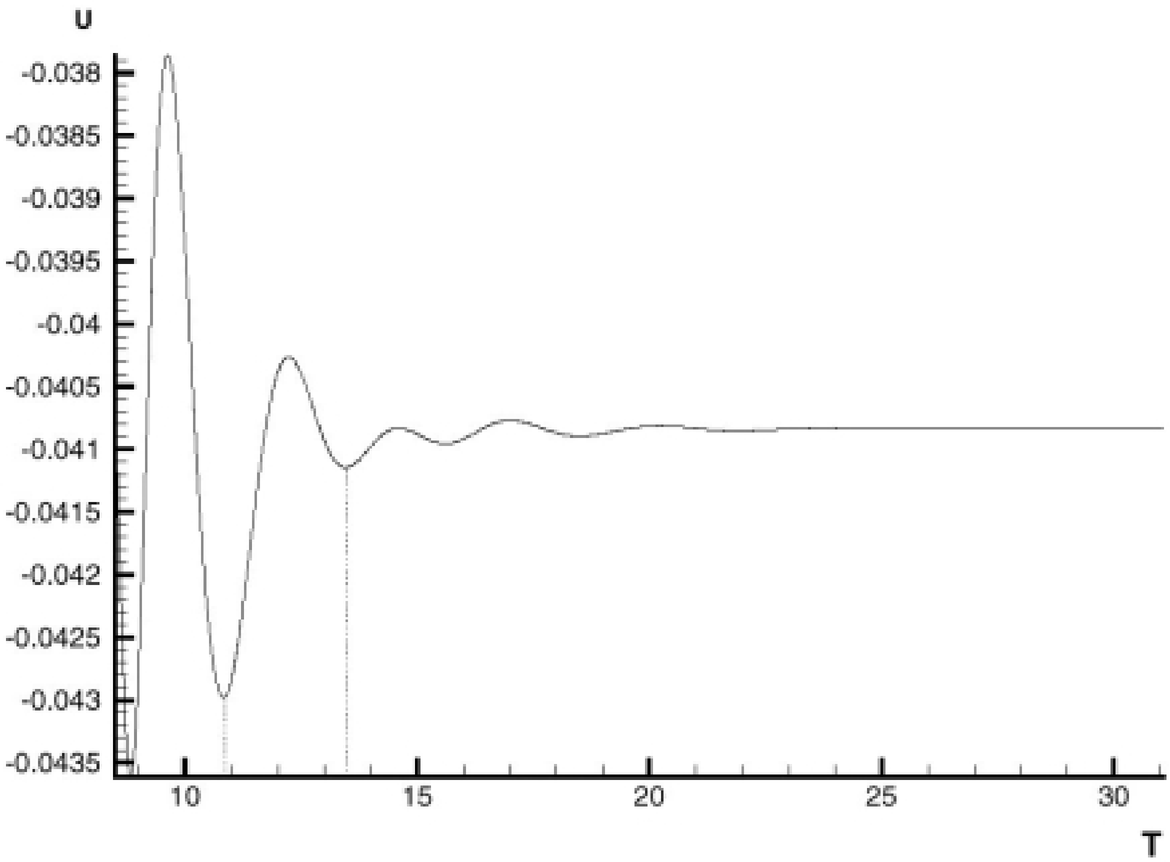

Figure 2: Characteristic oscillatory decay in time for the x-component of the velocity.

To analyze mechanisms for the onset of transition in radial flows, we introduce oscillatory perturbations at four symmetrically located positions at the entrance of the domain. Figure 2 shows the time evolution of the x-component of the velocity at a characteristic location in the domain (x = 1.186, y = 0.2511, z = 0.0531) for Re = 200. The flow field is converging towards

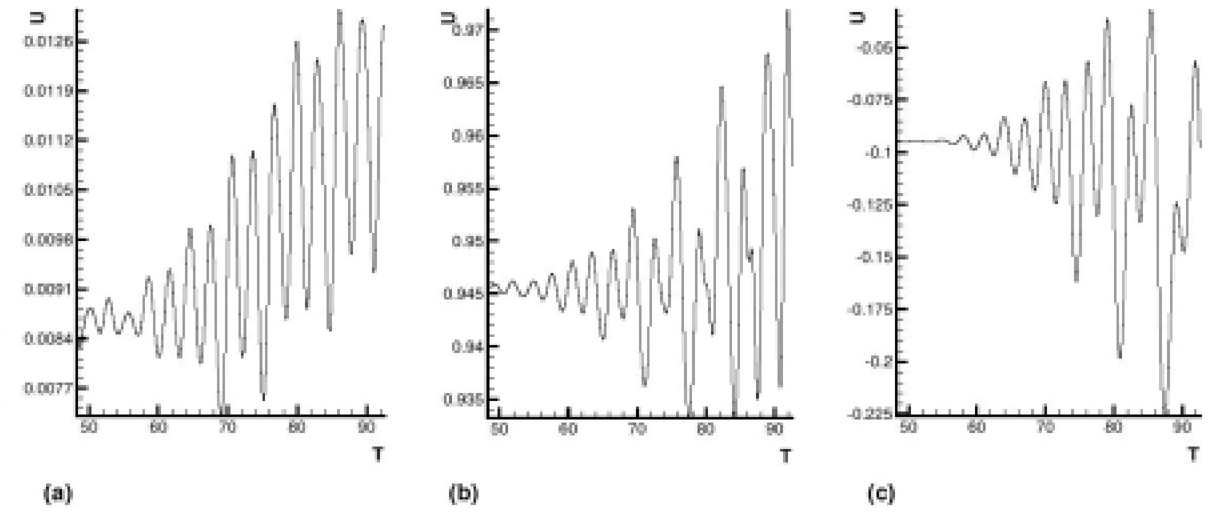

Figure 3: Time evolution of the three-dimensional flow at three characteristic locations.

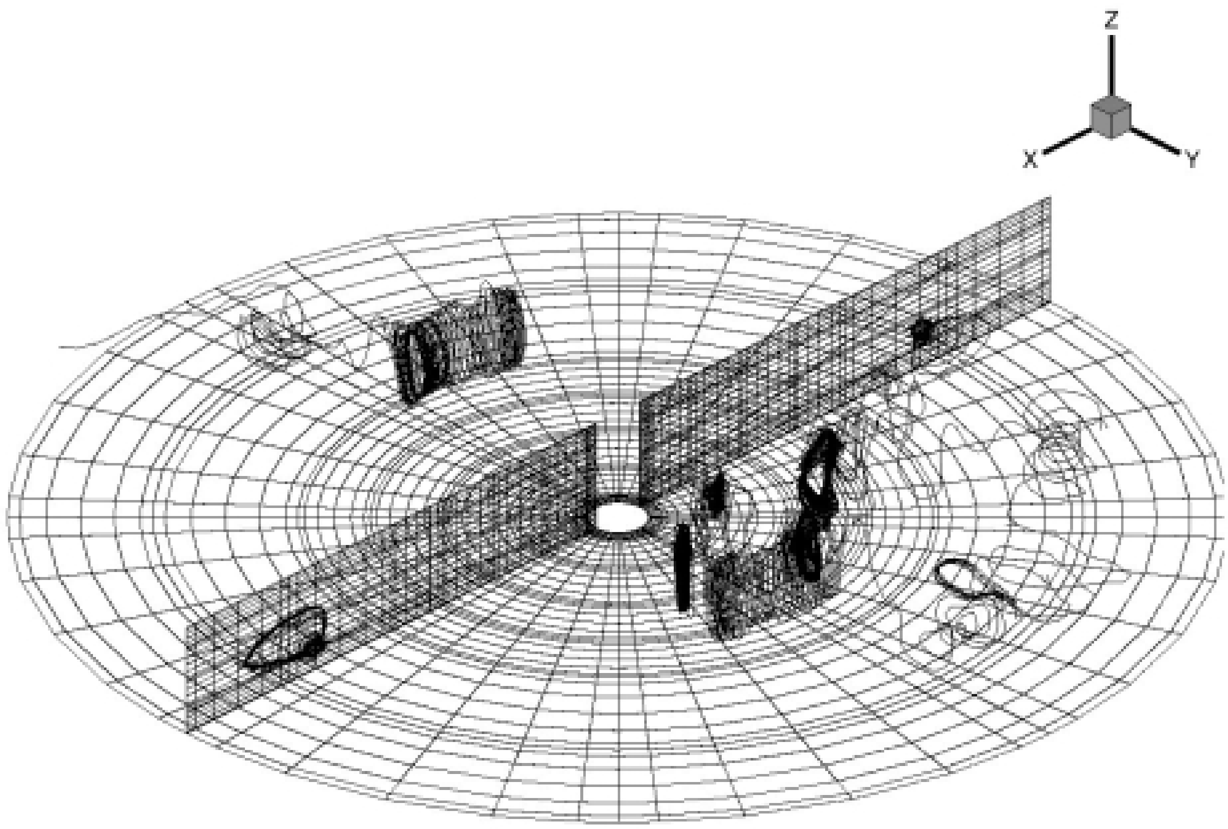

Figure 4: Instantaneous three-dimensional self-sustained disturbance streamtraces (Re = 200).

a steady-state basic flow. The distance between the dashed lines is associated to the frequency of the least stable modes. This frequency is used to define the forcing frequency of the oscillatory perturbations imposed at the entrance of the domain. Figure 3 depicts the time-evolution of the x-component of the velocity at three locations after *turning off* the oscillatory forcing at the entrance of the domain. We observe that the disturbance field does not decay in time. In fact, the disturbances grow in an oscillatory manner. We also notice a characteristic period of such self-sustained oscillations which is between the non-dimensional time values 2 and 3. The non-dimensional period (see Eq. (1)) associated to the least stable mode, which is used to characterize the oscillatory forcing at the inlet, is approximately equal to 2.4. Figure 4 presents the disturbance field streamtraces at a particular instant of time. We observe the presence of multiple vortices and their interactions by means of sinusoidal-like displacement of the fluid particles. The resultant disturbed flow field is characterized, when compared to Fig. 1, by a larger number of recirculation zones and increased mixing. This result is attractive for applications where the goal is to enhance heat transfer without demanding high pumping power (Amon, 1988). More significantly, it is the fact that this procedure may characterize a sub-critical mechanism for the onset of transition or the bifurcation of the flow to a self-sustained time-periodic behavior with higher amplitude.

4. CONCLUSIONS

Direct numerical simulations are performed to investigate the nonlinear stability of the three-dimensional radial flows, which are investigated and reported for the first time. Two types of perturbations are imposed into the flow: infinitesimally small perturbations and finite-amplitude oscillatory perturbations.

It is found that axisymmetric models fail to predict the flow physics for Re > 200, due to the existence of a secondary flow which cannot be captured by two-dimensional simulations. A characteristic frequency with non-dimensional period equal to 2.4 is obtained for the least stable modes corresponding to the time-evolution of the flow towards the steady-state solution. Oscillatory perturbations are imposed at the inlet of the three-dimensional flow domain with a frequency matching the least stable frequency of the decaying oscillatory perturbations. A significant increase in the circulation is observed, which improves mixing in the secondary flow. Subsequently, this forcing is turned off to investigate the response of the flow. This procedure destabilizes the flow with a characteristic non-dimensional period of the growing oscillations between 2 and 3. Additionally, this type of three-dimensional finite amplitude forcing seems to lead either to a self-sustained oscillatory flow or to a sub-critical onset of transition.

REFERENCES

Amon, C. H., 1988, Heat Transfer Enhancement and Three-Dimensional Transitional Flows by a Spectral-Fourier Method, Sc. D. Thesis, Massachusetts Institute of Technology.

Amon, C. H., 1993, Spectral Element-Fourier Method for Transitional Flows in Complex Geometries, AIAA Journal, 31, No. 1, pp. 42-48.

Canuto, C., Hussaini, M. Y., Quarteroni, A., Zang, T. A., 1988, Spectral Methods in Fluid Dynamics, Springer-Verlag, Berlin Heidelberg.

Drazin, P. G., Reid, W. H., 1981, Hydrodynamic Stability, Cambridge University Press.

Gottlieb, D., Orszag, S. A., 1977, Numerical Analysis of Spectral Methods: Theory and Applications, SIAM-CBMS, Philadelphia.

Langer, C. A., 1991, Separation, Bifurcation and Heat Transfer in Radial Flows, Master's Dissertation (in Portuguese), Mech. Eng. Dept., Federal University of Santa Catarina, Brazil.

Langer, C. A., Prata, A. T. and Ferreira, R. T. S., 1990, Separation and Bifurcation of Flows in Radial Diffuser, Annals of the 3rd National Meeting of Thermal Sciences (ENCIT), Itapema-SC, Brazil, Vol. 1, pp. 357-362.

Oliveira, J. C., 1992, Numerical Study of Hydrodynamic Stability in Radial Flows, Master's Dissertation (in Portuguese), Mech. Eng. Dept., Federal University of Santa Catarina, Brazil.

Oliveira, J. C., 1997, Evolution of Nonlinear Instabilities and Mathematical Analysis of Radial Flows. Ph. D. Thesis, Carnegie Mellon University.

Oliveira, J. C., Amon, C. H., 1996, Evolution of Nonlinear Instabilities in Radial Flows, ENCIT 96/LATCYM 96, Federal University of Santa Catarina, Florianopolis-SC-Brazil, Vol. 3, pp. 1803-1807.

Oliveira, J. C., Prata, A. T. and Langer, C. A., 1993, Numerical Study of the Hydrodynamic Stability in Radial Flows, Annals of the 12th Brazilian Congress of Mech. Eng., Brasilia, Brazil, Vol. 2, pp. 1021-1024.

Patera, A. T., 1984, Spectral Element Method for Fluid Dynamics, Journal of Computational Physics, Vol. 54, No. 3, pp. 468-488.

Peters, S., 1994, Bifurcations and Self-induced Oscillations in Fluid Flow through Radial Diffusers, Doctoral Thesis (in Portuguese), Mech. Eng. Dept., Federal University of Santa Catarina, Brazil.